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COMPLETE DIALLEL CROSS PLANS USING UNIT MATRIX

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ABSTRACT

Diallel cross is a mating scheme used by plant and animal breeders; and geneticists to investigate the genetic underpinnings of quantitative traits. In a complete diallel, all parents are crossed to make hybrids in all possible combinations. Complete diallel cross involve equal numbers of crosses of each of the p(p − 1)/2 distinct crosses among p inbred lines.

KEYWORDS: *Unit Matrix, Complete Diallel Crosses, General Combining Ability, and Efficiency Factor*

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INTRODUCTION

Diallel crossing is one type of mating design which was introduced by Schmidth (1919) and has been adopted in various situations by Comstock and Robinson (1952). It is useful method for conducting animal and plant breeding experiments, in particular with estimating the effects of the combining ability of lines. Griffing (1956) further developed appropriate models and methods of analysis. Diallel crosses in which all possible distinct crosses in pairs among the available lines are taken is called CDC. Sprague and Tatum (1942) defined the concept of general combining ability (GCA) and specific combining ability (SCA) effects. The GCA is the average performance of a line in hybrid combination, while SCA is the interaction between the i^{th} and j^{th} line effects. Several authors like, Kempthrone, (1957), Gilbert, (1958), Das and Sivaram, (1968), Agarwal and Das, (1990), Divecha and Ghosh, (1994) and Dey and Midha (1996), Das and Ghosh, (1999) etc. have developed several methods of construction of complete diallel crosses plan. Balanced Incomplete Block Design introduced by Yates (1936a) and developed by Fisher and Yates (1938a), Bose, (1939) were extensively used for the construction of CDC plans. They also discussed if the CDC plan is binary and connected then the design is optimal.. Dey and Midha, (1996), constructed CDC plans through triangular type PBIB design with two associate classes. Das and Ghosh (1999) developed balanced incomplete block diallel crosses designs. Ghosh and Biswas, (2003) concluded that the CDC plans obtained from two BIB designs with the same parameters using the Galois field are also universal optimal. Prasad, et.al (1999) constructed the universally optimal block designs for diallel crosses by using an application of a Galois field.

In this paper, we construct the CDC plan using unit matrix of size p. Moreover, we computed the efficiency factor of complete diallel crosses plans compare to completely randomized block design.

ANALYSIS OF COMPLETE DIALLEL CROSSES PLAN

Consider the Complete Diallel Crosses plan. Let Y be a n x 1 observational vector from a CDC plan, where $\frac{p(p-1)}{2}$ crosses are applied to n plots arranged in b blocks.

Let the linear model for the CDC plan is expressed as following:

$$Y = \mu 1_n + \Delta_1 g + \Delta_2 \beta + e \tag{1}$$

where Y denotes the n x 1 observational vector, μ is the general mean, 1_n denotes the n x 1 vector whose all elements are 1, g is the general combining ability (GCA) effects of vector p x 1 and β is block effects of vectors of p x 1, Δ_1 and Δ_2 are the corresponding design matrices of n x p and n x b respectively. That is, $(i, j)^{th}$ element of Δ_1 is 1, if the i^{th} observation is present in the j^{th} line and is zero otherwise. Similarly, the $(i, j)^{th}$ element of Δ_2 is 1, if the i^{th} observation comes from the j^{th} block and is zero otherwise, e is the random error vector components takes care of specific combining ability (SCA) and is un-assignable variation being distributed with mean zero and constant variance σ^2.

Gupta and Kageyama (1994) obtained the information matrix C

as $C_d = G_d - NK^{-1}N'$ (2)

Where, $\Delta_1'\Delta_1 = G_{dij}, S_{di} = g_{dij}.$ and $\Delta_1'\Delta_2 = N_d = (n_{dij})$

Where (n_{dij}) is the number of times i^{th} line occurs in the j^{th} blocks under the model. The reduced normal equation for GCA effects using design d is,

$$C_d \hat{g} = Q \tag{3}$$

Where, Q = T – $N_d K^{-1}$B; T is the vector of line totals and B is the vector of block totals. Now G_d can also be re-written as,

$$G_d = \begin{bmatrix} w_{di} & g_{ii} \\ g_{ii} & w_{di} \end{bmatrix} \tag{4}$$

Where, w_{di} denotes the diagonal elements as replication number of lines and g_{ii} denotes the off-diagonal elements as replication number of crosses

EFFICIENCY FACTOR OF CDC PLAN

Now, we adopt the Randomized Complete Block Design (RCBD), instead of CDC plan with 'r' blocks. So, total numbers of crosses are $\frac{r(v-2)}{2}$ and then the C-matrix of the randomized block design is obtained as,

$$C_d = r\,(v-2)\,[I_v - \frac{E_{vv}}{v}].$$

Hence, the variance of best linear unbiased estimator of any elementary contrast among GCA effects under RCBD is defined as,

$$Var\,(\hat{g}_i - \hat{g}_j) = \frac{2}{r(v-2)}\sigma^2, \text{ for all } I \neq j = 1, 2, ,\ldots, p.$$

Where, p is number of lines, r is number of times each cross occurs in CDC plan and σ^2 is error variance.

Now, in case of CDC plan, variance of best linear unbiased estimator of any elementary line contrast among GCA effect is expressed as,

$\mathrm{Var}\,(\hat{g}_i - \hat{g}_j) = \frac{2}{\theta}\sigma^2$, for all $I \neq j = 1, 2, ,\ldots, p$;

Where, θ denotes the non-eigen value of information matrix C with multiplicity (p – 1).

Now, efficiency factor of existing CDC plan compare to CRBD in r replication is defined as, Efficiency (E) = $\frac{\mathrm{Var}\,(\hat{g}_i - \hat{g}_j)_{CRBD}}{\mathrm{Var}\,(\hat{g}_i - \hat{g}_j)_{CDC}} = \frac{\frac{2}{r(p-2)}\sigma^2}{\frac{2}{\theta}\sigma^2}$.

METHOD OF CONSTRUCTION

We developed the method of construction of binary and non-binary complete diallel cross plan using the unit matrix of size p. The method of construction of binary and non-binary CDC plans is carried out separately in case 1 and case 2.

Case I: Binary complete diallel cross plan

In this case, we discuss the construction of binary CDC plan through example 1.

Example 1: Consider a unit matrix of size p = 7, where p is odd and is called number of lines. Delete the elements present in the first diagonal. That is, replace the element 1 by 0 at the diagonal place. Consider the residual matrix as an incidence matrix N of an incomplete block design. This is shown below as

$$M = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}, N = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

The blocks of the incomplete block design are following:

1	2	3	4	5	6
1	2	3	4	5	7
1	2	3	4	6	7
1	2	3	5	6	7
1	2	4	5	6	7
1	3	4	5	6	7
2	3	4	5	6	7

Now take the cross between two lines present in the same block. This gives three crosses per block. Select second block and take cross leaving one line such that no cross is repeated. Continue this process for all blocks. Since there is seven blocks, hence total number of crosses is twenty one, where each line occurs six times. Twenty one crosses arranged in seven blocks are following:

Table 1: Twenty one Crosses in Seven Blocks Blocks

Blocks	Number of Crosses								
1	1	x	2	3	x	4	5	x	6
2	1	x	3	2	x	4	5	x	7
3	1	x	4	2	x	3	6	x	7
4	1	x	5	2	x	6	3	x	7
5	1	x	6	2	x	7	4	x	5
6	1	x	7	3	x	5	4	x	6
7	2	x	5	3	x	6	4	x	7

From Table 1, it is obvious each of the seven lines do not occur in the seven blocks. However, each cross occurs one time in the entire plan. This clarify that CDC plan is conducted as incomplete block design.

The C matrix of the CDC plan is obtained from

$$C = \begin{bmatrix} 6 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 6 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 6 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 6 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 6 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 6 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 6 \end{bmatrix} - \begin{bmatrix} 6 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 6 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 6 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 6 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 6 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 6 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 6 \end{bmatrix} / 3$$

$$C = \begin{bmatrix} 12 & -2 & -2 & -2 & -2 & -2 & -2 \\ -2 & 12 & -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & 12 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & 12 & -2 & -2 & -2 \\ -2 & -2 & -2 & -2 & 12 & -2 & -2 \\ -2 & -2 & -2 & -2 & -2 & 12 & -2 \\ -2 & -2 & -2 & -2 & -2 & -2 & 12 \end{bmatrix} / 3$$

$C = \frac{14}{3}\left[I_7 - \frac{1}{7} E_{77}\right]$.

The non-zero Eigen value of C - matrix of the CDC plan is obtained as $\theta = \frac{14}{3}$ with multiplicity six.

The estimates of the i^{th} line effect is obtained from

$\hat{g}_i = \frac{1}{\theta} Q_i = \frac{3}{14} Q_i$. The variance of the i^{th} and j^{th} line effect for complete diallel cross plan is obtained from

$\mathrm{Var}\,(\hat{g}_i - \hat{g}_j) = \frac{2}{\theta}\sigma^2 = \frac{6}{14}\sigma^2$.

The well known result for the variance of best linear unbiased estimator of any elementary contrast among GCA effects of CRBD is obtained from

$\mathrm{Var}\,(\hat{g}_i - \hat{g}_j) = \frac{2}{r(p-2)}\sigma^2$, for all I ≠ j = 1, 2, ,…, p.

Since, each cross occurs once, so r = 1.

$\mathrm{Var}\,(\hat{g}_i - \hat{g}_j) = \frac{2}{5}\sigma^2$, as p = 7.

We can calculate the Efficiency factor of complete diallel cross plan from the expression

Efficiency (E) $= \frac{\mathrm{Var}\,(\hat{g}_i - \hat{g}_j)_{CRBD}}{\mathrm{Var}\,(\hat{g}_i - \hat{g}_j)_{CDC}} = \frac{\frac{2}{r(p-2)}\sigma^2}{\frac{2}{\theta}\sigma^2} = \frac{(2/5)\sigma^2}{(6/14)\sigma^2} = \frac{14}{15}$.

In this example, i^{th} line occurs only one time in the block hence, the CDC plan constructed using unit matrix is binary plan.

Now we explain the construction of non- binary CDC plan from the same unit matrix to compare the estimates and efficiency factor between binary and non-binary complete diallel cross plans. The method of construction is explained in case 2.

Case 2 Non-binary complete diallel cross plans

The method of construction of non-binary CDC plan is discussed as following. From case 1, we can observe that p lines are arranged in p blocks such that each block contains (p – 1) lines, and each line is repeated (p – 1) times. Select one block and take all possible crosses from (p – 1) lines and then keep them in one block. Each block has (p – 1) lines, thus we have $\frac{(p-1)(p-2)}{2}$ all possible number of crosses per block, where each cross is distinct. However, number of lines is repeated in the same block, thus the constructed CDC plan is non-binary. Again, there are p blocks and hence total number of crosses are $\frac{p(p-1)(p-2)}{2}$. Using this method, we can construct a non-binary CDC plan with p lines arranged in p blocks such that each block contains $\frac{(p-1)(p-2)}{2}$ crosses, where each line is repeated (p – 1)(p – 2) times and each cross is repeated (p – 2) times.

Example 2: Consider the same example as discussed in case 1. The seven blocks of this block design are following:

1	2	3	4	5	6
1	2	3	4	5	7
1	2	3	4	6	7
1	2	3	5	6	7
1	2	4	5	6	7
1	3	4	5	6	7
2	3	4	5	6	7

Select one block, take all possible crosses between two lines of the same block. In this way, we get 15 crosses per block. We proceed to continue the same procedure for all the blocks. Total number of 105 crosses is arranged in 7 blocks, where each line occurs 30 times whereas each cross is repeated 5 times. The CDC plan is shown in Table 2

Table 2: 105 Crosses in 7 Blocks

Bl	Crosses														
1	1x2	1x3	1x4	1x5	1x6	2x3	2x4	2x5	2x6	3x4	3x5	3x6	4x5	4x6	5x6
2	1x2	1x3	1x4	1x5	1x7	2x3	2x4	2x5	2x7	3x4	3x5	3x7	4x5	4x7	5x7
3	1x2	1x3	1x4	1x6	1x7	2x3	2x4	2x6	2x7	3x4	3x6	3x7	4x6	4x7	6x7
4	1x2	1x3	1x5	1x6	1x7	2x3	2x5	2x6	2x7	3x5	3x6	3x7	5x6	5x7	6x7
5	1x2	1x4	1x5	1x6	1x7	2x4	2x5	2x6	2x7	4x5	4x6	4x7	5x6	5x7	6x7
6	1x3	1x4	1x5	1x6	1x7	3x4	3x5	3x6	3x7	4x5	4x6	4x7	5x6	5x7	6x7
7	2x3	2x4	2x5	2x6	2x7	3x4	3x5	3x6	3x7	4x5	4x6	4x7	5x6	5x7	6x7

In Table 2, the i^{th} line occurs more than one time, so the constructed CDC plan is non-binary design.

The C matrix of the non-binary CDC plan is obtained from

$$C = \begin{bmatrix} 30 & 5 & 5 & 5 & 5 & 5 & 5 \\ 5 & 30 & 5 & 5 & 5 & 5 & 5 \\ 5 & 5 & 30 & 5 & 5 & 5 & 5 \\ 5 & 5 & 5 & 30 & 5 & 5 & 5 \\ 5 & 5 & 5 & 5 & 30 & 5 & 5 \\ 5 & 5 & 5 & 5 & 5 & 30 & 5 \\ 5 & 5 & 5 & 5 & 5 & 5 & 30 \end{bmatrix} - \begin{bmatrix} 150 & 125 & 125 & 125 & 125 & 125 & 125 \\ 150 & 125 & 125 & 125 & 125 & 125 & 125 \\ 150 & 125 & 125 & 125 & 125 & 125 & 125 \\ 150 & 125 & 125 & 125 & 125 & 125 & 125 \\ 150 & 125 & 125 & 125 & 125 & 125 & 125 \\ 150 & 125 & 125 & 125 & 125 & 125 & 125 \\ 150 & 125 & 125 & 125 & 125 & 125 & 125 \end{bmatrix} / 15$$

$$C = \begin{bmatrix} 300 & -50 & -50 & -50 & -50 & -50 & -50 \\ -50 & 300 & -50 & -50 & -50 & -50 & -50 \\ -50 & -50 & 300 & -50 & -50 & -50 & -50 \\ -50 & -50 & -50 & 300 & -50 & -50 & -50 \\ -50 & -50 & -50 & -50 & 300 & -50 & -50 \\ -50 & -50 & -50 & -50 & -50 & 300 & -50 \\ -50 & -50 & -50 & -50 & -50 & -50 & 300 \end{bmatrix} /15$$

$C = \frac{350}{15}\left[I_7 - \frac{1}{7}E_7\right]$.

For this example, the non-zero eigen value of C - matrix is $\theta = \frac{350}{15}$ with multiplicity 6.

The estimate of the i^{th} line effect is obtained from

$\hat{g}_i = \frac{1}{\theta} Q_i = \frac{15}{350} Q_i$, and variance of the i^{th} and j^{th} line effect under CDC plan is obtained from

$\text{Var}(\hat{g}_i - \hat{g}_j) = \frac{2}{\theta}\sigma^2 = \frac{30}{350}\sigma^2$.

The variance of best linear unbiased estimator of any elementary line effects under CRBD is obtained as,

$\text{Var}(\hat{g}_i - \hat{g}_j) = \frac{2}{r(p-2)}\sigma^2$, for all $I \neq j = 1, 2, ,\dots, p$.

In this case each cross occurs five times, so r = 5, so we have,

$\text{Var}(\hat{g}_i - \hat{g}_j) = \frac{2}{25}\sigma^2$,

The Efficiency factor of CDC plan is obtained from the expression

$\text{Efficiency (E)} = \frac{\text{Var}(\hat{g}_i - \hat{g}_j)_{CRBD}}{\text{Var}(\hat{g}_i - \hat{g}_j)_{CDC}} = \frac{\frac{2}{25}\sigma^2}{\frac{30}{350}\sigma^2}$

$= \frac{14}{15}$.

CONCLUSIONS

WE observed that the efficiency factor for both the binary and non-binary CDC plan is same, though the estimates of the GCA effects in both cases are different. The estimates of GCA effect in case of binary CDC plan is $\frac{3}{7}\sigma^2$, whereas the estimates of GCA effect in case of non-binary CDC plan is $\frac{3}{35}\sigma^2$. This is obvious the estimates of GCA effect in case of binary CDC plan is greater than the estimates of GCA effect in case of non-binary CDC plan. Because of this reason we prefer to suggest the use of binary CDC plan.

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